



Comparison of Flow Characteristics and Energy Dissipation in Vertical Drop Structures with Aerated and Non-Aerated Flow

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Abstract— Vertical drop structures, owing to their simple construction and effective performance, are common hydraulic features employed in irrigation networks, erodible waterways, and watersheds for managing flow energy. In these structures, the occurrence of a hydraulic jump downstream, a critical parameter for energy loss akin to most energy-consuming structures, significantly influences the overall waste of energy. The dissipation of energy transpires due to the flow's impact on the pond, along with alterations in flow lines and the turbulence generated in the pond beneath. Another crucial component contributing to energy loss is the jet, referred to as "jet energy loss" in this study. Properly sizing the slope-breaking structure in accordance with the hydraulic characteristics of the flow is crucial, with special attention to mitigating jet energy loss. The incorporation of continuous slope breakers in the flow path can notably reduce operational costs. This research investigates flow parameters by constructing vertical drop structures at two heights, namely 5.15 cm and 5.25 cm. The study focuses on flow conditions with non-aerated jets and compares the findings with similar studies conducted in aerated flow conditions by researchers such as Rajaratnam, White, Rand, and Moore. The results indicate variable dissipation levels dependent on the number of slope breakers, with higher dissipation observed in non-aerated states compared to aerated states. For the non-aerated state, the dissipation ranges from approximately 6.6% of total flow energy for D_n equal to 0.36 to 57% for D_n above 0.003.

Keywords— Drop Structure, Energy Dissipation, Drop Structure Number, Ventilation, Froud Number.

I. INTRODUCTION

Vertical drops are among the most common hydraulic structures used in irrigation networks, wastewater collection systems, water treatment systems, etc., serving various purposes such as reducing kinetic energy, providing ventilation, and increasing dissolved oxygen flow (Chanson, 2004b; Guven, 2011). Most of the energy is dissipated based on the hydraulic characteristics of the flow, being lost both upstream and downstream during the occurrence of a hydraulic jump. Consequently, a significant portion of the flow energy is consumed by the fall of the flow jet into the basin, followed by mixing within it (Chatelier et al., 2011; Li et al., 2011).

The objective of this research is to explore existing theories for estimating energy dissipation related to the jet stream and other hydraulic characteristics of vertical drops. Initially, a concise review was conducted on methods proposed by different researchers concerning the hydraulics of drop flow in aerated jet mode. Subsequently, based on laboratory measurements obtained from physical models under aerated conditions, the results were compared

with methods proposed by researchers for aerated states and hydraulic relationships for non-aerated flow were derived (Jin & Lant, 2004, 2004; Morchain et al., 2000).

II. FLOW HYDRAULICS

Many researchers have studied the characteristics of flow in drop structures over the past years. Most of the research is based on hydraulic characteristics in vertical drops, specifically related to the sub-critical flow regime (upstream) in the aeration jet mode. However, only a limited amount of research has been conducted on the supercritical regime. The term "supercritical flow regime" in the upstream of the drop structure implies that the channel above the drop structure has a supercritical bottom slope. Figure (1) illustrates the hydraulic characteristics of the flow through the drop structure as follows (Akan, 2006; Chanson, 2004a; Stefanovski & Dimirovski, 2001):

H= The height of the slope, y_u = Normal flow depth above hand, y_b = The depth of the flow is exactly above the edge of the drop, y_p = Flow depth in the pool under the jet, Φ = The angle of impact of the jet on the pond below it, V = The average speed of the jet before entering the pond, y_1 = The depth of the current before the jump, y_2 = Secondary depth jump, L_p = The length of the pool under the jet, L_j = Hydraulic jump length.

Using the principles of the quantity of movement of the relationship (1) to determine the ratio (y_p/y_u) presented (Mobley, 2001).

$$\frac{y_p}{y_u} = \sqrt{\left(\frac{y_1}{y_u}\right)^2 + 2Fr_u^2 \left(\frac{y_u}{y_1}\right) - (2Fr_u^2 + 1)} \quad (1)$$

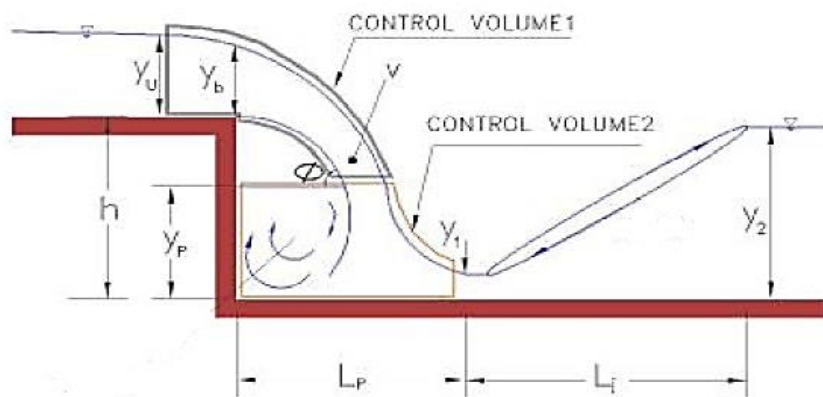


Figure 01: Introduction of the vertical drop and its different components

In this equation, Fr_u represents the Froude number of the flow upstream of the drop structure. Depending on the shape of the gradually variable profile upstream of the drop structure, when the above relation is applied in subcritical flow conditions, the depth y_u becomes equal to the critical depth of the flow (y_c), resulting in a Fr_u value of 1. If equation (1) is used for supercritical flow, where the theoretical effect of the drop structure on the water level profile is limited, y_u can be considered as the normal depth of the flow upstream of the drop structure.

A method presented by Chamani & Beirami, based on trial and error, is employed to determine the hydraulic characteristics of the slope breaker. The relationships used in this method are as follows (Chamani & Beirami, 2002).

$$\cos \phi = \frac{(-1 + \sqrt{1 + \frac{8A}{\sqrt{2}}B})}{2}$$
$$A = 2Fr_u^2 / (1 + 2Fr_u^2)$$
$$B = \sqrt{(1 + 0.5Fr_u^2 + \left(\frac{h}{y_u}\right) - \frac{y_p}{y_n}}$$

(2)

$$V_m = \frac{1}{2} (1 + \cos \phi) \sqrt{2g[y_u(1 + 0.5Fr_u^2) + h - y_p]} \quad (3)$$

$$y_1^3 - \frac{1}{2g} (V_m^2 + 2gy_p)y_1^2 + \frac{q^2}{2g} = 0 \quad (4)$$

In the above relationships, V_m represents the horizontal component of the flow velocity at the point of entry into the basin below the drop structure, and q denotes the discharge per unit width of the flow. In Chamani and Birami's proposed method, the calculation of ϕ from equation (2) is based on the given problem information, including y_u , Fr_u , and h . Assuming a value for y_p , the calculation of V_m is derived from equation (3). By using q , y_p , and V_m , the determination of y_1 is made possible through equation (4). The accuracy of the assumed value of y_p is then assessed by substituting the value of y_1 into equation (1).

Chamani and Birami note that the above equations and method can be used for both supercritical and subcritical flow regimes. To verify the accuracy of the proposed method, they conducted experiments on vertical drops with heights of 0.2 and 0.4 meters in a flume measuring 0.4 meters in width and 9.5 meters in length.

Their findings indicate that, for a flow unit of width (q), an increase in the Froude number of the flow results in a decrease in the relative energy dissipation of the slope breaker, the depth of the flow before the jump, and the water depth in the pool under the jet. Additionally, for a specific Froude number, an increase in q leads to a decrease in $\Delta H/h_1$, while the depths y_1 and y_p increase. In summary, the values predicted by Chamani and Birami's methods exhibit good agreement with the measured values. However, it should be noted that, according to their conclusion, the assumptions made in the proposed method, including ignoring air penetration and floor shear stress, could be reasons for any observed differences.

In general, the values predicted by the methods of Chamani and Beirami demonstrate good agreement with the measured values. However, it should be noted that, according to their conclusion, the assumptions made in the proposed method, such as ignoring air penetration and floor shear stress, may account for observed differences (Chamani & Beirami, 2002).

Moore, through experiments on vertical drop structures with a sub-critical flow regime upstream, concluded that the amount of flow energy dissipation significantly depends on the relative height of the drop structure ($h/y_c = Dn^{1/3}$) (Moore, 1943).

Moore states that the relative energy dissipation of the current, $\Delta H/H_1$, changes from zero to 0.53 with an increase in the h/y_c ratio. In 1943, White presented equation (5) to estimate the depth y_1 before the jump, which is utilized in the USBR standard for the design of vertical drop structures.

$$\frac{y_1}{y_c} = \frac{\sqrt{2}}{1.061 + \sqrt{1.5 + \frac{h}{y_c}}} \quad (5)$$

which can be used to obtain the flow energy in section (1) H_1 from the following equation:

$$\frac{H_1}{y_c} = \frac{\sqrt{2}}{1.061 + \sqrt{1.5 + \frac{h}{y_c}}} + 0.25(1.061 + \sqrt{1.5 + \frac{h}{y_c}})^2 \quad (6)$$

On the other hand, the energy of the flow upstream of the drop structure (H_t) in the subcritical regime can be expressed as follows:

$$H_t = h + 1.5y_c \quad (7)$$

Later, by correcting White's theory, Gill proposed the following relationship to obtain the depth of y_1 (Gill, 1979):

$$\frac{y_1}{y_c} = \left[\frac{(1 + \cos \phi)^2}{2} \left(\frac{h}{y_c} + 1.5 - \frac{y_p}{y_c} \right) + 2 \left(\frac{y_p - y_1}{y_c} \right) \right]^{-0.5} \quad (8)$$

Rand has also conducted studies on vertical drop s, which led to the following relationships.

$$\frac{y_1}{y_c} = 0.54 \left(\frac{y_c}{h} \right)^{0.275} \quad (9)$$

$$\frac{y_2}{h} = 1.66 \left(\frac{y_c}{h} \right)^{0.81} \quad (10)$$

$$\frac{y_p}{h} = \left(\frac{y_c}{h} \right)^{0.66} \quad (11)$$

$$\frac{L_p}{h} = 4.3 \left(\frac{y_c}{h} \right)^{0.81} \quad (12)$$

$$L_j = 6.9(y_2 - y_1) \quad (13)$$

Chamani and Rajaratnam, by ignoring the phenomenon of air penetration and floor shear stress and using the relation of the amount of movement for two control volumes 1, 2 in figure (1) obtained the relations (14) and (15) respectively.

$$pqV \cos \phi + \frac{1}{2} \omega y_p^2 = pqV_1 + \frac{1}{2} \omega y_1^2 \quad (14)$$

$$\frac{1}{2} \omega y_c^2 + pqV_c = pqV \cos \phi \quad (15)$$

Also, using the energy relation, the following relation can be derived.

$$h + 1.5y_c = \frac{v^2}{2g} + y_p \quad (16)$$

By integrating the relations (14) and (15), the unknowns \emptyset, V will be removed and using the following empirical relation presented by them, the unknown parameters of the problem V, y_1, y_p will be obtained.

$$\frac{y_p}{h} = 1.107 \left(\frac{y_c}{h} \right)^{0.719} \quad (17)$$

III. RESEARCH METHODOLOGY

The practical stages of this research were conducted in the hydraulic laboratory of the School of Water Science Engineering at Pakistan Institute of Engineering & Applied Sciences, Islamabad. Two slope drops, measuring 15.5 cm and 25.5 cm in height, were constructed using Plexiglas. These models were then installed in a glass flume with a width of 25 cm and a length of 10 m, featuring a false floor. Water flow was pumped from the laboratory's underground tank to a 4.5-meter-high tank, with the flow rate entering the flume regulated by a control valve. Upon entering the flume and passing over the models, a hydraulic jump formed at the flume's end. The water was released from a control valve that managed the trace depth and jump position. Downstream of the flume, a triangular spillway with a 53-degree vertical arm measured the flow rate. Water level measurements were taken using a point gauge equipped with a vernier, providing an accuracy of 0.1 mm. Aligning with the research objectives, all measurements were conducted under conditions of upstream subcritical flow and non-aerated conditions. To minimize errors caused by air penetration effects on y_1 measurement in the high-speed current section before the jump, the conjugate depth was initially measured, and then the depth of y_1 was determined using the jump relationship. Kamayab Moghaddam et al in their study on energy loss in a stepped spillway with inclined steps, (Kamyab Moghaddam et al., 2022). investigated Chamani and Rajaratnam's energy equation. The test results, when compared with Chansun's correlation and Bernoulli's energy equation, exhibited substantial agreement. Employing a consistent methodology, in this study the test results also compared with the equations proposed by Chamani and Rajaratnam, as well as White. This uniform approach enhances the reliability and comprehensiveness of the process. Considering that in the non-aerated flow state, the air pressure under the flow jet is lower than atmospheric pressure and is unknown, the relation (1) cannot be used for this current state. Consequently, an approach was taken to establish a relationship between observational data and compare them with experimental data and prediction methods provided by different researchers.

IV. RESULTS AND DISCUSSION

The statistical analysis of the data measured on the physical models of the current research showed the relationship between the $y_p/h, y_1/h, \Delta H/H_1, y_c/h$ in conditions of subcritical and non-aerated flow respectively as follows:

$$\frac{y_p}{h} = 35.495 \left(\frac{y_c}{h} \right)^3 - 11.957 \left(\frac{y_c}{h} \right)^2 + 3.48 \left(\frac{y_c}{h} \right)^1, n=12, r^2=0.97 \quad (18)$$

$$\frac{y_1}{h} = 0.512 \left(\frac{y_c}{h} \right)^{1.208}, n=19, r^2=0.99 \quad (19)$$

$$\frac{\Delta H}{H_t} = 0.762 e^{3.625 \frac{y_c}{h}}, n=19, r^2=0.87 \quad (20)$$

$$\frac{L_p}{h} = 2.05\left(\frac{y_c}{h}\right)^{0.63}, n=19, r^2=0.85 \quad (21)$$

In their investigation of energy loss in a stepped spillway with inclined steps, Kamyab Moghaddam et al. (Kamyab Moghaddam et al., 2022) compared dissipation energy loss for each slope inclination among all examined slopes and fitted a curve between different points. This study also utilizes their approach to maintain consistency and ensure comparability in the analytical methodology. In Figures 2, 3, 4, and 5, the dimensionless ratio of Y_c/h is plotted against Y_p/h , Y_1/h , and L_p/h , respectively, in the upper subcritical and non-aerated flow regime. The results are compared with those of other researchers in the aerated flow state. As observed, the changes of Y_p/h compared to Y_c/h exhibit a small difference at low values of Y_c/h in the non-aerated state, as compared to the values in the aerated state. This difference increases with the increase of Y_c/h , such that for values of Y_c/h exceeding 0.32, y_p is practically not formed under the jet. According to diagram (3), the difference of Y_1/h in the non-aerated state compared to the aerated state is also insignificant at the same Y_c/h . However, it should be noted that due to the very high speed of the flow in the section before the jump, even small changes in y_1 among the hydraulic parameters can significantly alter the relative energy dissipation.

Furthermore, it appears that the relationship proposed by White in 1943 shows a Y_1/h value higher than the measured values. This suggests that White's method tends to overestimate the relative amount of energy dissipation. The relative amount of energy dissipation compared to Y_c/h in the unventilated state, as depicted in diagram (4), surpasses the experimental data or predictions made by other researchers in all cases, except for the proposed method by White.

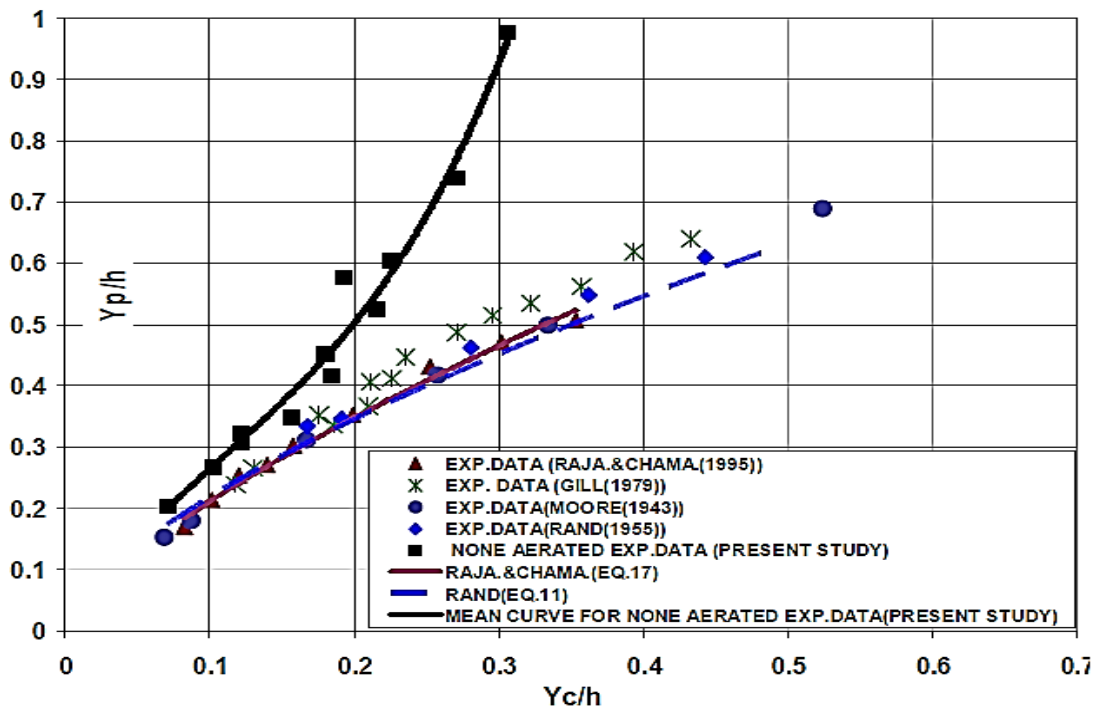


Figure 02: Changes of y_p/h , y_c/h in the non-aerated flow state according to the laboratory data of the current research and its comparison with the results of other researches in the aerated state

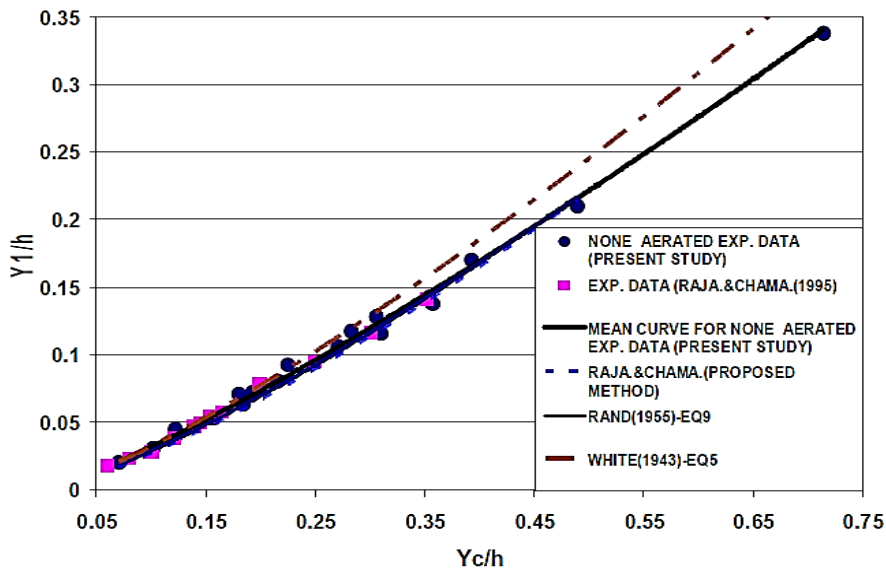


Figure 03: Changes of y_c/h , y_1/h in the non-aerated flow state according to the laboratory data of the present research and its comparison with the results of other studies in the aerated state

Of course, this issue was predictable due to the aeration effect under the jet, which reduces the impact angle of the jet on the basin (Φ). When comparing the values of $\Delta H/h_t$ in the non-aerated state with the aerated state in the proposed method of Chamani and Rajaratnam, it is evident that the difference in values at Y_c/h greater than 0.6 is almost zero. However, as Y_c/h decreases to around 0.07, the difference increases to about 18%. It should be noted that the use of vertical drop structures with unventilated flow is justified due to the potential for instability caused by the ingress and egress of unsustainable airflow, especially in applications where changes in the water level above are crucial. Such use may be considered unjustifiable.

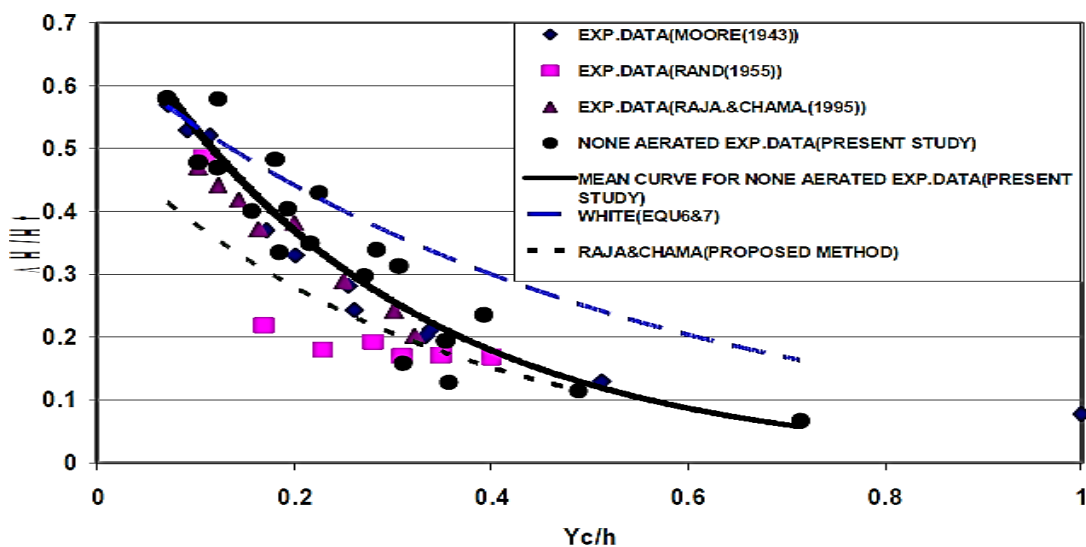


Figure 04: changes of $\Delta H/h_1$ compared to y_c/h in non-aerated flow state according to the laboratory data of the current research and its comparison with the results of other researches in the ventilated state

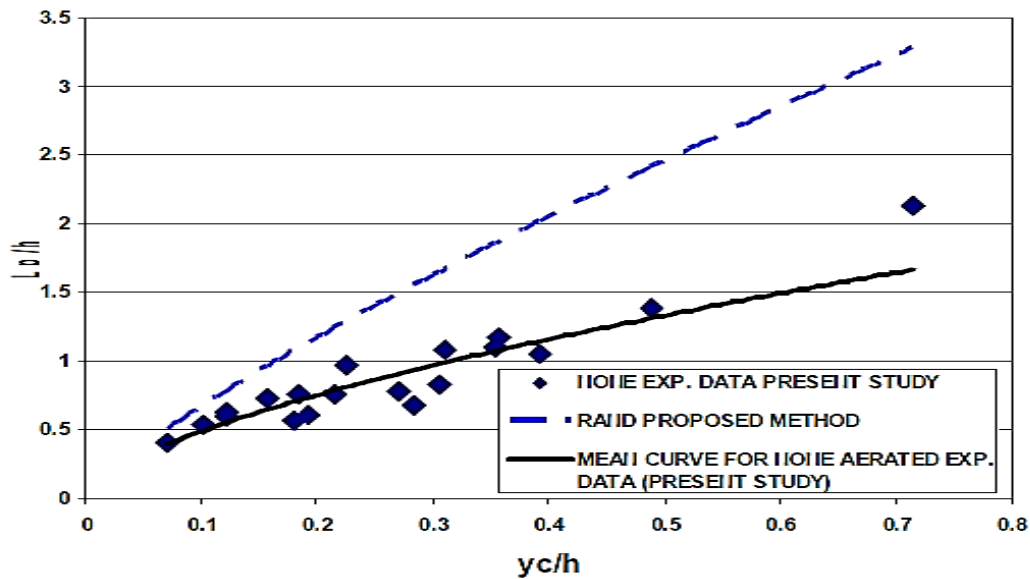


Figure 05: changes of L_p/h compared to yc/h in non-aerated flow state according to research laboratory data present and its comparison with the results of other researches in aerated state

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